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Geometry, D-Branes and $N = 1$ Duality in Four Dimensions with Product Gauge Groups

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Abstract

We study $N = 1$ dualities in four dimensional supersymmetric gauge theories by D 6-branes wrapping around 3-cycles of Calabi-Yau threefolds in type IIA string theory. We consider the models involving $SU(N_{c1}) \times SU(N_{c2})$ product gauge group without or with adjoint matter in terms of geometrical realization of the configuration of D 6-branes wrapped 3-cycles. We also find simple geometric descriptions for the triple product gauge group $SU(N_{c1}) \times SU(N_{c2}) \times SU(N_{c3})$ interpreted recently by Brodie and Hanany in the context of D brane configurations together with NS 5-branes. Their introduction of semi infinite D 4-branes appears naturally by looking at the flavor group in the dual theory. We generalize to product of arbitrary number of gauge groups.

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1 Introduction and Geometrical Picture

String theory interprets many nontrivial aspects of four dimensional $N = 1$ supersymmetric field theory by T-duality on the local model for the compactification manifold. There are two approaches to describe this local model.

One is to consider the local description as purely geometric structure of compactification manifold together with D-branes wrapping around cycles [1, 2, 3]. The compactification of F-theory on elliptic Calabi-Yau(CY) fourfolds from 12 dimensions leads to $N = 1$ supersymmetric field theories in four dimensions. It has been studied in [1], for the case of pure $SU(N_c)$ Yang-Mills gauge theory, that the gauge symmetry can be obtained in terms of the structure of the D 7-brane worldvolume. By adding D 3-branes and bringing them near the complex 2-dimensional surface, the local string model gives rise to matter hypermultiplets in the fundamental representation with pure $SU(N_c)$ Yang-Mills theory [2]. Moreover, Seiberg's duality [4] for the $N = 1$ supersymmetric field theory can be mapped to a T-duality exchange of the D 3-brane charge and D 7-brane charge. For the extension to $SO(N_c)$ and $Sp(N_c)$ gauge theories [5, 6] coupled to matter, the local string models are type IIB orientifolds with D 7-branes on a curved orientifold 7-plane [3].

The other is to interpret $N = 1$ duality for $SU(N_c)$ gauge theory as D-brane description together with NS 5-branes in a flat geometry [7] according to the approach of [8]. Extension of this to the case of $SO(N_c)$ and $Sp(N_c)$ gauge theories with flavors was presented in [9] by adding an orientifold 4-plane. The generalization to the construction of product gauge groups $SU(N_{c1}) \times SU(N_{c2})$ with matter fields is given in [10, 11] by suspending two sets of D 4-branes between three NS 5-branes. It was also considered in [12] a brane configuration which gives rise to electric and magnetic theories for $SO(N_{c1}) \times Sp(N_{c2})$ product gauge group. Very recently, the dual aspects of $N = 1$ supersymmetric $SO(10)$ gauge theory with arbitrary numbers of spinors and vectors have been found in [13] in the context of field theory.

Following the idea of [2, 3], Ooguri and Vafa have observed in [14] that $N = 1$ duality can be embedded into type IIA string theory with D 6-branes, partially wrapped around three cycles of CY threefold, filling four dimensional spacetime. They discussed what happens to the wrapped cycles and studied the relevant field theory results when a transition in the moduli of CY threefolds occurs. Furthermore, they reinterpreted the configuration of D-branes in the presence of NS 5-branes [7] as purely classical geometrical realization.

It is natural to ask how a number of known field theory dualities which contain *additional* field contents arise from two different approaches we have described so far. It

was obtained in [15], according to the approach of [14], that one can generalize the work of [14] to various models, consisting of one or two 2-index tensors and some fields in the defining representation (fundamental representation for $SU(N_c)$ and $Sp(N_c)$, vector representation for $SO(N_c)$), presented earlier by many authors [16] and study its geometric realizations by wrapping D 6-branes about 3-cycles of CY threefolds. On the other hand, by introducing a multiple of coincident NS 5-branes and NS' 5-branes, it was argued [17] that $SU(N_c)$ gauge theory with one or two adjoints superfields in addition to the fundamentals can be obtained. Moreover, $SO(N_c)$ gauge group with an adjoint field and N_f vectors was described in terms of a multiple of coincident NS 5-branes and a single NS' 5-brane with orientifold 4-plane (similarly $Sp(N_c)$ with a traceless antisymmetric tensor and N_f flavors by adding orientifold 6-plane). See also the relevant papers [18] analyzing the brane configurations associated with field theories in various dimensions. Recently [19] we applied the approach of [14] to the various models presented by Brodie and Strassler [20], consisting of D_{k+2} superpotential with SU, SO and Sp gauge groups along the line of [15]. We discussed a large number of representations for a field Y, but with a field X always in the adjoint (symmetric) [antisymmetric] representation for $SU(SO)[Sp]$ gauge groups where the superpotential $W = \mathrm{Tr} X^{k+1} + \mathrm{Tr} XY^2$.

In this paper, we go one step further. We find geometric descriptions for the multiple product gauge groups for $SU(N_c)$ without or with adjoint matter. We compare our results with those obtained by Brodie and Hanany [10] who used a different approach and reinterpret their semi infinite D 4-branes as the condition for flavor gauge group in the dual picture.

We will review the main geometrical setup of [14, 15] in the remaining part of this section. Let us start with the compactification of type IIB string theory on the CY threefold leading to $N = 2$ supersymmetric field theories in 4 dimensions. Suppose we have various D 3-branes wrapping around a set of three cycles of CY threefold. It is known [21] that whenever the integration of the holomorphic 3-form on the CY threefold around three cycles takes the form of parallel vectors in the complex plane, such a D 3-brane configuration allows us to have a BPS state. Then after we T-dualize the 3-spatial directions of three torus T^3 we obtain type IIA string theory with D 6-branes, partially wrapping around three cycles of CY threefold and filling 4 dimensional spacetime. We end up with $N = 1$ supersymmetric field theories in 4 dimensions.

The local model of CY threefold can be described by [22, 14] five complex coordinates x, y, x', y' and z satisfying the following equations:

$$x^2 + y^2 = \prod_i (z - a_i), \quad x'^2 + y'^2 = \prod_j (z - b_j)$$

where each of C^*'s is embedded in (x, y)-space and (x', y')-space respectively over a generic point z. This describes a family of a product of two copies of one-sheeted hyper-

boloids in (x,y)-space and (x',y')-space respectively parameterized by the z-coordinates. For a fixed z away from a_i and b_j there exist nontrivial S^1's in each of C^*'s corresponding to the waist of the hyperboloids. Notice that when $z=a_i$ or $z=b_j$ the corresponding circles vanish as the waists shrink. Then we regard 3 cycles as the product of $S^1 \times S^1$ cycles over each point on the z-plane, with the segments in the z-plane ending on a_i or b_j. When we go between two a_i's (b_j's) without passing through b_j (a_i) the 3 cycles sweep out $S^2 \times S^1$. On the other hand, when we go between a_i and b_j the 3 cycle becomes S^3. We will denote the 3-cycle lying over between a_i and b_j by $[a_i, b_j]$ and also denote other cycles in a similar fashion. In next section we will describe our main results by exploiting this geometrical setup. We will start with by writing down the configurations of ordered points in the real axis of z-plane to various models we are concerned with.

2 Two gauge groups: $SU(N_{c1}) \times SU(N_{c2})$

Let us first briefly review the product gauge group $SU(N_{c1}) \times SU(N_{c2})$ in the brane configuration picture. As discussed in [10], in this case it does not suffice to have NS 5 branes oriented at 0 degrees (i.e. in (x^4,x^5) plane) and 90 degrees (i.e. in (x^8,x^9) plane). We cannot have parallel NS 5-branes for a theory which has N=1 supersymmetry, so for more than two NS 5-branes, they need to appear at different angles in (x^4,x^5,x^8,x^9). To be more specific we repeat here an argument of [23]. One defines the complex planes $z_1 = x^4 + ix^8$ and $z_2 = x^5 + ix^9$. In the case of only two NS 5-branes , it is enough to have branes in (x^4,x^5) direction (which is the one given by Im $z_1 =$ Im $z_2 = 0$ and in (x^8,x^9) direction (which is the one given by Re $z_1 =$ Re $z_2 = 0$). If one has more than two NS 5-branes, a configuration which preserves exactly $N=1$ supersymmetry is the one in which all the NS 5-branes are rotated at different angles in (x^4,x^5,x^8,x^9) direction. This can be understood by applying a rotation $z_1 \to e^{i\theta_i} z_1$ and $z_2 \to e^{-i\theta_i} z_2$ to the i-th NS 5-brane.

Using this argument, it was considered in [10] three NS 5-branes A, B and C from left to right with N_{c1} D 4-branes between A and B and N_{c2} D 4-branes between B and C. A is rotated at an angle θ_1, B is in (x^4,x^5) direction and C is rotated at an angle θ_3. They also inserted N_{f1} D 6-branes parallel with A and N_{f2} D 6-branes parallel with C. This is the electric theory. The magnetic theory was obtained by moving B and C to the right of A and in the final picture there were $\widetilde{N}_{c1} (= 2N_{f2} + N_{f1} - N_{c2})$ D 4-branes between C and B and $\widetilde{N}_{c2} (= 2N_{f1} + N_{f2} - N_{c1})$ D 4-branes between B and A.

Now we go to the geometric approach and we study how the duality is obtained. In the geometric approach there are two possibilities to view the theory, one is to take the original geometric picture in terms of singularities of CY 3-folds and the other one is

obtained by a T-duality applied to the original picture and is almost identical with the description of the brane configuration approach. The last one differs from the description of the brane configuration approach in the sense that the role of the N_f D 6-branes is played by a NS 5-brane. To be more precise, in brane configuration, we consider two NS 5-branes A and B with N_c D 4-branes connecting them and N_f D 6-branes which intersect the D 4-branes. On the other hand in the geometric approach, after a T-duality, we have three NS 5-branes A, B and C, N_c D4 branes between A and B and N_f D4-branes between B and C where C has to be parallel with A.

Let us introduce now the electric theory in the geometric approach. We study supersymmetric Yang-Mills theory with gauge group $SU(N_{c1}) \times SU(N_{c2})$, with $N_{f1}(N_{f2})$ flavors in the fundamental representation of the first(second) gauge group. Besides we have the fields X and Y in the $(\mathbf{N_{c1}}, \overline{\mathbf{N}}_{\mathbf{c2}})$ and $(\overline{\mathbf{N}}_{\mathbf{c1}}, \mathbf{N_{c2}})$ representations of the product gauge groups. The theory has a superpotential W = Tr $(XY)^2$ for unique NS 5-branes localised at different singularity points and W = Tr $(XY)^{k+1}$ for k NS 5-branes localised at every singularity point.

2.1) The theory with superpotential W = Tr $(XY)^2$

We start by wrapping D 6-branes around 3-cycles of a Calabi-Yau threefold in type IIA string theory where the 3-cycle is either $S^2 \times S^1$ or S^3. We may regard (x^6, x^7) as real and imaginary parts of z coordinate. The geometry is viewed as a $C^* \times C^*$ bundle over the z-plane with (x, y) coordinates in the first C^* and (x', y') coordinates in the second C^*. On the real axis of z-plane, we consider 5 singular points denoted by (a_1, c_2, b, a_2, c_1) from left to right. We shall later see (after T-duality) why we make this choice. Suppose that the first C^* degenerates at a_1, a_2, c_1 and c_2 and the second C^* degenerates at b. In the fibre above b, the second C^* is in (45) direction while the fibre above a_1 and a_2 is rotated at an angle θ_1 in (4589) directions and the fibre above c_1 and c_2 is rotated at an angle θ_2 in (4589) directions. The cycles $[a_i, b], [c_i, b]$ are S^3 cycles and the cycles $[a_1, a_2], [c_1, c_2]$ and $[a_i, c_j]$ are $S^2 \times S^1$ cycles. We wrap N_{c1} D 6-branes on the cycle $[a_1, b]$, N_{c2} D 6-branes on the cycle $[b, c_1]$, N_{f1} D 6-branes on the cycle $[b, a_2]$ and N_{f2} D 6-branes on the cycle $[c_2, b]$. The field theory we end up with is given by an $N = 1$ gauge theory with gauge group $SU(N_{c1}) \times SU(N_{c2})$ with flavor groups $SU(N_{f1})$ and $SU(N_{f2})$ which now are gauge groups. In order to make them global we need to push a_2 and c_2 to infinity. To do that we have to interchange the positions of a_2 and c_1 on one side and the positions of a_1 and c_2 on the other side.

When we make this change something interesting happens. To see that let us perform a T-duality in (x^4, x^5, x^8, x^9) directions. We now have five NS 5-branes, one at each of the previous five points. The ones at a_1 and a_2(those at c_1 and c_2) are parallel and are rotated at an angle $\theta_1(\theta_2)$ in (4589) directions. The NS 5-branes at a_2 and c_2 play the role of the N_{f1} and N_{f2} D 6-branes respectively in the brane configuration picture where

by moving N_{f1} D 6-brane from right to left with respect to a NS 5-brane, N_{f1} D 4-brane will appear to the left of the N_{f1} D6-brane. Our claim here is that when we move the NS 5-branes sitting at c_2 from right to left with respect to the one sitting at a_1, the same amount of D 4-branes will appear! This is a new phenomenon compared with the previous considered cases and is the main idea of our present work. It has to occur in order to match the results obtained in field theory and in brane configuration.

Going back (by T-duality) to the original picture, this means that when we move c_2 to the left of a_1 then N_{f2} supplementary D 6-branes are wrapped on $[c_2, a_1]$. The same argument tells us that when we move a_2 to the left of c_1, then N_{f1} supplementary D 6-branes are wrapped on $[c_1, a_2]$. We thus have the following configuration (from left to right): $2N_{f2}$ D 6-branes are wrapped between c_2 and a_1, $(N_{c1} + N_{f2})$ D 6-branes are wrapped between a_1 and b, $(N_{f1} + N_{c2})$ D 6-branes are wrapped between b and c_1 and $2N_{f1}$ D 6-branes are wrapped between c_1 and a_2.

Now we want to move to other point in the moduli of the CY threefold and end up with a configuration where again the degeneration points are along the real axis in the z-plane, but the order is changed to (c_2, c_1, b, a_1, a_2). The transition is similar to the one made for simple groups, in the sense that the point b changes positions with both a_1 and c_1. This is the reason for taking the original picture the way we took it. First we push c_1 up along the imaginary direction. Then $(N_{f1} + N_{c2})$ D 6-branes are wrapped on $[b, a_2]$ and $(N_{f1} - N_{c2})$ D 6-branes are wrapped on $[c_1, a_2]$. We continue to move c_1 along the negative real axis and insert it between c_2 and a_1. At this moment the $(N_{f1} - N_{c2})$ D 6-branes which were wrapped on $[c_1, a_2]$ decompose to $(N_{f1} - N_{c2})$ D 6-branes wrapped on $[c_1, a_1]$, $(N_{f1} - N_{c2})$ D 6-branes wrapped on $[a_1, b]$ and $(N_{f1} - N_{c2})$ D 6-branes wrapped on $[b, a_2]$. So, after the first transition, there are $2N_{f2}$ D 6-branes wrapped on $[c_2, c_1]$, $(2N_{f2} + N_{f1} - N_{c2})$ D 6-branes on $[c_1, a_1]$, $(N_{c1} + N_{f1} + N_{f2} - N_{c2})$ D 6-branes wrapped on $[a_1, b]$ and $2N_{f1}$ along $[b, a_2]$. Similarly we push b between c_1 and a_1. When we raise it along the imaginary axis there are $(N_{c1} + N_{f1} + N_{f2} - N_{c2})$ D 6-branes wrapped on $[a_1, a_2]$ and $(N_{f1} - N_{f2} - N_{c1} + N_{c2})$ D 6-branes wrapped on $[b, a_2]$. By moving the point b along the negative real axis and inserting it between c_1 and a_1, we obtain the final configuration of points ordered as (c_2, c_1, b, a_1, a_2) with $2N_{f2}$ D 6-branes wrapped on $[c_2, c_1]$, $(2N_{f2} + N_{f1} - N_{c2})$ D 6-branes wrapped on $[c_1, b]$, $(2N_{f1} + N_{f2} - N_{c1})$ D 6-branes wrapped on $[b, a_1]$ and $2N_{f1}$ D 6-branes wrapped on $[a_1, a_2]$. This picture describes the magnetic theory with a gauge group $SU(2N_{f2} + N_{f1} - N_{c2}) \times SU(2N_{f1} + N_{f2} - N_{c1})$ with $N_{f2}(N_{f1})$ flavors in the fundametal representation of the first(second) gauge group. In order to obtain global flavor group, we move a_2 and c_2 to infinity. Besides we have singlets which correspond to the mesons of the original electric theory and interact with the dual quarks through the superpotential in the magnetic theory. The magnetic description coincides with the one obtained in field theory and in brane configuration.

2.2) The theory with superpotential W = Tr $(XY)^{k+1}$

In the brane configuration, the electric theory corresponds to k NS 5-branes with the same orientation as A connected to a single B NS 5-brane by N_{c1} D4-branes from the left and k NS 5-branes with the same orientation as C connected to a single B NS 5-brane from the right by N_{c2} D 4-branes. In the geometrical picture, after a T-duality, this would mean that instead of having one NS 5-brane at a_1 we have k NS 5-branes and instead of having one NS 5-brane at c_1 we have k NS 5-branes. In order to handle the problem we consider instead of k NS 5-branes at one point, k points with one NS 5-brane at each point. This way of seeing things was introduced in [15] for the group $SU(N_c)$ with adjoint matter. The electric theory is a $SU(N_{c1}) \times SU(N_{c2})$ gauge group with $N_{f1}(N_{f2})$ flavors in the fundamental representation of the first(second) gauge group. Besides we have the same fields X and Y as in the previous subsection with the superpotential W $=$ Tr $(XY)^{k+1}$.

For simplicity, let us consider first the case $k = 2$. On the real axis we will have from left to right: $(a_{11}, a_{12}, c_2, b, a_2, c_{11}, c_{12})$, where instead of $a_1(c_1)$ we have $a_{11}, a_{12}(c_{11}, c_{12})$. Using the idea of [15, 19], we wrap N_1 D 6-branes on $[a_{11}, b]$, N_2 D 6-branes on $[a_{12}, b]$, N_3 D 6-branes on $[b, c_{11}]$ and N_4 on $[c_{12}, b]$ such that $N_1 + N_2 = N_{c1}$ and $N_3 + N_4 = N_{c2}$. Besides we wrap N_{f1} D 6-branes on $[b, a_2]$ and N_{f2} D 6-branes on $[b, c_2]$. As before, we move c_2 to the left of a_{11}, a_{12} and a_2 to the right of c_{11} and c_{12}.

To apply the observation made in the previous subsection, perform first a T-duality in (x^4, x^5, x^8, x^9) directions. In the T-dual language, to move c_2 to the left of a_{11}, a_{12} means to move the NS 5-brane sitting at c_2 to the left with respect to the NS 5-branes sitting at a_{11} and a_{12}. When c_2 passes a_{11}, N_{f2} supplementary D 6-branes are wrapped on $[c_2, a_{11}]$ and when it passes a_{12} another N_{f2} D 6-branes appear! Going back to the original geometrical picture, we have $3N_{f2}$ D 6-branes wrapped on $[c_2, a_{11}]$, $(2N_{f2} + N_1)$ D 6-branes wrapped on $[a_{11}, a_{12}]$ and $(N_{f2} + N_{c1})$ D 6-branes wrapped on $[a_{12}, b]$. In the limit $a_{11} \to a_{12}$, the only difference between this configuration and the one of the previous subsection is that we have $3N_{f2}$ D 6-branes wrapped on $[c_2, a_{11} = a_{12} = a_1]$ instead of $2N_{f2}$. The same thing appears when we move a_2 to the right of c_{11}, c_{12}. In the limit $c_{11} \to c_{12}$ the result differs from the fact that we have $3N_{f1}$ branes on $[c_{11} = c_{12} = c_1, a_2]$ instead of $2N_{f1}$. We have the following configuration: $3N_{f2}$ D 6-branes are wrapped on $[c_2, a_{11} = a_{12}]$, $(N_{c1} + N_{f2})$ D 6-branes are wrapped on $[a_{11} = a_{12}, b]$, $(N_{f1} + N_{c2})$ D 6-branes are wrapped on $[b, c_{11} = c_{12}]$ and $3N_{f1}$ D 6-branes are wrapped between $[c_{11} = c_{12}, a_2]$. We want to move to another point of moduli space of the CY threefold by changing the positions of $a_{11} = a_{12} = a_1$ and $c_{11} = c_{12} = c_1$. We push c_1 and b on the imaginary direction and move them along the negative real direction to the right of a_1 to obtain the magnetic theory. The final configuration is (c_2, c_1, b, a_1, a_2) with $3N_{f2}$ D 6-branes wrapped on $[c_2, c_1]$, $(3N_{f2} + 2N_{f1} - N_{c2})$ D 6-branes wrapped on $[c_1, b]$, $(3N_{f1} + 2N_{f2} - N_{c1})$ D 6-branes wrapped on $[b, a_1]$ and $3N_{f1}$ D 6-branes wrapped on $[a_1, a_2]$. This gives a field theory with the gauge group $SU(3N_{f2}+2N_{f1}-N_{c2}) \times SU(3N_{f1}+2N_{f2}-N_{c1})$ with $N_{f1}(N_{f2})$ flavors in the fundamental

of the second(first) gauge group.

The generalisation to arbitrary k becomes now obvious. Instead of $a_1(c_1)$ we take k singular points $a_{11}, a_{12}, \cdots, a_{1k}(c_{11}, c_{12}, \cdots, c_{1k})$. The points a_2, b, and c_2 will remain in a single copy. Now wrap N_i on $[a_{1i}, b]$ for each i between indices 1 and k and M_j on $[b, c_{1j}]$ for each j between indices 1 and k with the condition $\sum_{i=1}^{k} N_i = N_{c1}$ and $\sum_{j=1}^{k} M_j = N_{c2}$. Also we wrap N_{f1} on $[b, a_2]$ and N_{f2} on $[c_2, b]$. When we displace $a_{11}, \cdots, a_{1k}$ to the right of c_2, for every transition, N_{f2} supplementary D 6-branes will appear. Eventually we have $(k+1)N_{f2}$ D 6-branes wrapped on $[c_2, a_{11}]$, etc. The same thing happens when we move $c_{11}, \cdots, c_{1k}$ to the left of a_2 and we finish with $(k+1)N_{f1}$ branes wrapped on $[c_{11}, a_2]$. Now we identify $a_{11} = \cdots = a_{1k} = a_1$ and $c_{11} = \cdots = c_{1k} = c_1$. The configuration is similar with the one of the previous subsection, the only difference being the fact that we have $(k+1)N_{f1}$ D 6-branes on $[c_2, a_1]$ and $(k+1)N_{f2}$ D 6-branes on $[c_1, a_2]$ instead of $2N_{f1}$ and $2N_{f2}$ respectively. We change again the positions of b and c_1 from the right to the left of a_1 to obtain the magnetic description. In terms of field theory this has a gauge group $SU((k+1)N_{f2} + kN_{f1} - N_{c2}) \times SU((k+1)N_{f1} + kN_{f2} - N_{c1})$ with $N_{f2}(N_{f1})$ flavors in the fundamental of the first(second) gauge group. The result is the same as the one obtained in [10] by changing the brane configuration.

2.3) $SU(N_{c1}) \times SU(N_{c2})$ with adjoint matter

Now we introduce adjoint matter. In the brane configuration picture, this corresponds to k NS 5-branes with the same orientation as A connected by N_{c1} D 4-branes with k NS 5-branes with the same orientation as B which are connected to k NS 5-branes with the same orientation as C by N_{c2} D 4-branes. The adjoint fields X_1 and X_2 correspond to the motion of the k NS 5-branes in (x^4, x^5, x^8, x^9) directions. In the geometrical picture, after a T-duality, this means that we have k NS 5-branes at a_1, k NS 5-branes at b and k NS 5-branes at c_1. Using the ideas of [15, 19], we apply the same methods as in section 2.2. Again we take instead of k NS 5-branes at one point, k points with one NS 5-brane at each point. This means that instead of single singular points a_1, b, c_1 we take k copies of them(i.e. we take as singular points $a_{11}, \cdots, a_{1k}, b_1, \cdots, b_k, c_{11}, \cdots, c_{1k}$ with one NS 5-brane at each point). The electric theory has the gauge product $SU(N_{c1}) \times SU(N_{c2})$, with $N_{f1}(N_{f2})$ flavors in the fundamental representation of the first(second) gauge group. Besides we have flavors in the adjoint representations of each component of the product gauge group. Denote $X_1(X_2)$ the adjoint of the first(second) gauge group. Then the superpotential is: $W = \mathrm{Tr}X_1^{k+1} + \mathrm{Tr}X_2^{k+1} + \mathrm{Tr}X_1YX + \mathrm{Tr}X_2YX + \rho_1\mathrm{Tr}X_1 + \rho_2\mathrm{Tr}X_2$ where ρ_i are Lagrange multipliers necessary in order to impose the tracelessness condition for X_1 and X_2. The power k can be any positive integer. If we take k to be one, the problem reduces to the one treated in the section 2.1. So we will take k greater than 2.

Let us take $k = 2$. the configuration looks like this: $(a_{11}, a_{12}, c_2, b_1, b_2, a_2, c_{11}, c_{12})$.

Note that a_2 and c_2 are always in a single copy. We now wrap N_1 D 6-branes on $[a_{11}, b_1]$, N_2 D 6-branes on $[a_{12}, b_2]$, N_{f2} on $[c_2, b_1]$, N_{f2} on $[c_2, b_2]$, N_3 on $[b_1, c_{11}]$, N_4 on $[b_2, c_{12}]$, N_{f1} on $[b_1, a_2]$, and N_{f1} on $[b_2, a_2]$, with the condition that $N_1+N_2 = N_{c1}, N_3+N_4 = N_{c2}$. If we take the limit $b_1 \to b_2$, we have now $2N_{f2}$ D 6-branes wrapped on $[c_2, b_1 = b_2]$ and $2N_{f1}$ D 6-branes on $[b_1 = b_2, c_1]$. Again the configuration looks very much alike the previous ones.

We now move a_{11}, a_{12} to the right of c_2 and c_{11}, c_{12} to the left of a_2. In this case, we apply again the observation about the supplementary branes. In the present case the transition looks as in section 2.2 but we start with $2N_{f2}$ D 6-branes wrapped on $[c_2, b_1 = b_2]$ and $2N_{f1}$ D 6-branes wrapped on $[b_1 = b_2, a_2]$. We obtain $4N_{f2}$ D 6-branes wrapped on $[c_2, a_{11}]$, $(3N_{f2}+N_1)$ wrapped on $[a_{11}, a_{12}]$, $(2N_{f2}+N_{c1})$ wrapped on $[a_{12}, b]$, $(2N_{f1} + N_{c2})$ wrapped on $[b, c_{11}]$, $(3N_{f1} + N_4)$ wrapped on $[c_{11}, c_{12}]$ and $4N_{f2}$ wrapped on $[c_{22}, a_2]$.

We now move to other point in the moduli of the CY threefold and end up with the magnetic theory. Taking the limit $a_{12} \to a_{11}$ and $c_{12} \to c_{11}$, we have the following starting configuration $(c_2, a_{11} = a_{12} = a_1, b, c_{11} = c_{12} = c_1, a_2)$, with $4N_{f2}$ D 6-branes wrapped on $[c_2, a_1]$, $(2N_{f2} + N_{c1})$ wrapped on $[a_1, b]$, $(2N_{f1} + N_{c2})$ wrapped on $[b, c_1]$ and $4N_{f1}$ wrapped on $[c_1, a_2]$. To get the magnetic description we move again b and c_1 from the right to the left of a_1. The procedure is the same as before and we obtain the magnetic theory with the gauge group $SU(4N_{f2}+2N_{f1}-N_{c2})\times SU(4N_{f1}+2N_{f2}-N_{c1})$. Our result again agrees with the result of [24, 10] for $k = 2$.

To generalise to the case of arbitrary k, we have to take k copies of all the singular points a_1, b, c_1. Compared to section 2.2, the only difference is the appearance of k copies of b. When all these copies coincide, we will have kN_{f2} D 6-branes wrapped on $[c_2, b]$ and kN_{f1} wrapped on $[b, a_1]$. The electric theory has the following configuration of singular points $(c_2, a_{11} = \cdots = a_{1k} = a_1, b_1 = \cdots = b_k = b, c_{11} = \cdots = c_{1k} = c_1, a_2)$, with $2kN_{f2}$ D 6-branes wrapped on $[c_2, a_1]$, $(kN_{f2}+N_{c1})$ D 6-branes wrapped on $[a_1, b]$, $(kN_{f1}+N_{c2})$ D 6-branes wrapped on $[b, c_1]$ and $2kN_{f1}$ D 6-branes wrapped on $[c_1, a_2]$. The magnetic theory is obtained by changing the position of b and c_1 from right to left with respect to a_1. The procedure is the same as the one performed in section 2.1 and we obtain the magnetic theory with the gauge group $SU(2kN_{f2}+kN_{f1}-N_{c2})\times SU(2kN_{f1}+kN_{f2}-N_{c1})$, result which again agrees with the one obtained in field theory and in brane configuration for any value of k.

3 The product of more than two SU groups

To complete the analogy with the results obtained by field theory methods and by brane configurations, we consider now the case of a product of more than two SU groups. One of the most important features of our construction in this case is that we were able to give an explanation in the geometric picture for the semi-infinite D 4-branes that appear in the brane configuration picture. In [10], these semi-infinite D 4-branes were neccesary in order to match the field theory calculation for the dual gauge group. In our case, we make an assumption based on the dualities in geometrical picture. That is, for $SU(N_c)$ case, the flavor group in the dual theory is determined by the D 6-branes which are wrapped on $[a_1, a_2]$. For $SU(N_{c1}) \times SU(N_{c2})$ case, the flavor group in the dual picture is determined by the D 6-branes which are wrapped on $[a_1, a_2]$ and $[c_1, c_2]$. We thus observe that the flavor group in the dual theory is always determined by D 6-branes wrapped on pair of singular points. By pair of singular points we understand here points where, after a T-duality, the NS 5-branes are parallel. This is a main observation and we will use it to obtain the gauge group in the magnetic theory. We will also show that this condition is completely equivalent to the requirement of introducing semi-infinite D 4-branes!

3.1) $SU(N_{c1}) \times SU(N_{c2}) \times SU(N_{c3})$

Consider the theory with gauge group $SU(N_{c1}) \times SU(N_{c2}) \times SU(N_{c3})$, with $N_{f1}(N_{f2})[N_{f3}]$ flavors in the fundamental representation of the first(second)[third] SU group. Besides, we have a field X in the $(\mathbf{N_{c1}}, \overline{\mathbf{N}}_{\mathbf{c2}})$ representation and its conjugate $\widetilde{X}$ and a field Y in the $(\mathbf{N_{c2}}, \overline{\mathbf{N}}_{\mathbf{c3}})$ representation and its conjugate $\widetilde{Y}$. To truncate the chiral ring, we add the superpotential: $W = \frac{1}{2}\mathrm{Tr}(X\widetilde{X})^2 + \mathrm{Tr}X\widetilde{X}Y\widetilde{Y} - \frac{1}{2}\mathrm{Tr}(Y\widetilde{Y})^2$. Now we want to see how this looks like in the geometric picture. Consider the singular points to be, from left to right $(a_1, c_2, b, d_2, a_2, c_1, d_1)$. We wrap N_{c1} D 6-branes on $[a_1, b]$, N_{c2} D 6-branes on $[b, c_1]$, N_{c3} D 6-branes on $[c_1, d_1]$, N_{f1} D 6-branes on $[b, a_2]$, N_{f2} D 6-branes on $[c_2, b]$ and N_{f3} D 6-branes on $[d_2, c_1]$ which is the electric theory.

We do want to go to the magnetic theory by moving to another point in the moduli of the CY threefold. First move c_2 and d_2 to the left of a_1 in order to be able to send them to infinity and to obtain a global flavor group. When moving c_2 to the left of a_1, there will be N_{f2} *additional* D 6-branes wrapped on $[c_2, a_1]$. So there will be $2N_{f2}$ D 6-branes wrapped on $[c_2, a_1]$. Now move the point d_2. When d_2 passes b, there are N_{f3} *additional* D 6-branes wrapped on $[d_2, b]$. When d_2 passes a_1 there are another N_{f3} additional D 6-branes wrapped on $[d_2, a_1]$ which one has to add to the previous additional N_{f3}. Insert now d_2 between c_2 and a_1. Then we have $2N_{f2}$ D 6-branes wrapped on $[c_2, d_2]$, $(2N_{f2} + 3N_{f3})$ wrapped on $[d_2, a_1]$ and $(2N_{f3} + N_{c1} + N_{f2})$ wrapped on $[a_1, b]$. We now move a_2 to the right of c_1 and d_1. When a_2 passes c_1, there are N_{f1} additional D 6-branes wrapped on $[c_1, a_2]$. When a_2 passes d_1 there another additional N_{f1} D 6-branes giving

$2N_{f1}$ additional D 6-branes wrapped on $[d_1, a_2]$. So, when we move a_2 to the right of c_1 and d_1, we have $(N_{f1}+N_{c2}+N_{f3})$ D 6-branes wrapped on $[b, c_1]$, $(2N_{f1}+N_{c3})$ wrapped on $[c_1, d_1]$ and $3N_{f1}$ wrapped on $[d_1, a_2]$.

We now move to the magnetic theory. To do that, we move b, c_1 and d_1 to the left of a_1, obtaining the configuration $(c_2, d_2, d_1, c_1, b, a_1, a_2)$ on the real axis. We first move the point d_1. When we push d_1 along the imaginary direction, there are $(2N_{f1}+N_{c3})$ D 6-branes wrapped directly on $[c_1, a_2]$ and $(N_{f1}-N_{c3})$ on $[a_2, d_1]$. By moving d_1 along the negative real axis and inserting it between d_2 and a_1, we obtain $(3N_{f3}+2N_{f2}+N_{f1}-N_{c3})$ D 6-branes wrapped on $[d_1, a_1]$, $(2N_{f3}+N_{f1}+N_{f2}+N_{c1}-N_{c3})$ on $[a_1, b]$, $(2N_{f1}+N_{f3}+N_{c2}-N_{c3})$ on $[b, c_1]$ and $3N_{f1}$ on $[c_1, a_2]$. When we push c_1 along the imaginary direction, there are $(N_{f1}-N_{c2}-N_{f3}+N_{c3})$ D 6-branes wrapped on $[c_1, b]$ and $(2N_{f1}+N_{f3}+N_{c2}-N_{c3})$ D 6-branes wrapped on $[b, a_1]$. The point c_1 is inserted between d_1 and a_1 so there will be $(2N_{f3}+2N_{f2}+2N_{f1}-N_{c2})$ D 6-branes wrapped on $[c_1, a_1]$. The last step is to move b between c_1 and a_1. By making this we obtain that there are $(3N_{f1}+N_{f2}+N_{f3}-N_{c1})$ D 6-branes wrapped on $[b, a_1]$ and $3N_{f1}$ D 6-branes wrapped on $[a_1, a_2]$. The magnetic picture that we have obtained is identical with the one obtained in [10]. But it does not match exactly the one obtained by field theory methods.

In [10], they have obtained the field theory result by introducing semi-infinite D 4-branes. To apply their method, we first make a T-duality transformation. We have NS 5-branes at each of the singular points. The NS 5-branes at a_2, c_2, d_2 play the role of the N_{f1}, N_{f2} and N_{f3} D 6-branes in brane configuration picture. Consider that we did not send a_2, c_2 and d_2 to infinity. Insert $2N_{f2}$ D 4-branes between the NS 5-branes sitting at c_1 and c_2. The conservation of the linking number for the NS 5-branes sitting at c_2 requires $2N_{f2}$ semi-infinite D 4-brane at its left. The conservation of the linking number for the NS 5-brane sitting at c_1 requires $2N_{f2}$ D 4-branes at its right. They will combine with the $(2N_{f2}+2N_{f3}+2N_{f1}-N_{c2})$ D 4-brane existent between the NS 5-branes sitting at c_1 and b to give $(4N_{f2}+2N_{f3}+2N_{f1}-N_{c2})$ D 4-branes which is the right result for the middle gauge group obtained by field theory methods. Now we have to add $2N_{f2}$ D 4-branes to the right of the NS 5-brane sitting at the point b. N_{f2} of them combine with the rest of D 4-branes between the NS 5-branes sitting at the point b and a_1 to give $(3N_{f1}+2N_{f2}+N_{f3}-N_{c1})$ D 4-brane which is the right result for the last part of the product gauge group. To conserve the linking number, we need to add N_{f2} semi-infinite D 4-branes to the right of the NS 5-branes sitting at b and a_1.

Let us see what is the correspondence in the original geometric picture. Before any insertion, we have $3N_{f3}$ D 6-branes wrapped on $[d_2, d_1]$ and $2N_{f2}$ D 6-branes wrapped on $[c_2, d_1]$. As we discussed at the beginning of this section, we put the condition that in the dual picture, the flavor group is given by D 6-branes wrapped only on the pair singularity points. We have cycles of pairs $[c_1, c_2]$, $[a_1, a_2]$ and $[d_1, d_2]$. The point b does

not have a pair, so we might have D 6-branes wrapped for example on $[b, a_2]$.

After the transition, we have D 6-branes wrapped on $[a_1, a_2]$ and $[d_1, d_2]$ but we also have $2N_{f2}$ D 6-branes wrapped on $[c_2, d_1]$ which do not satisfy our condition. We want to extend these to D 6-branes wrapped on $[c_2, c_1]$. To completely understand what is the process, we need to go back and forth between the original geometrical picture and its T-dual. The insertion of $2N_{f2}$ D 4-branes between the NS 5-branes sitting at c_1 and c_2 means here to connect the D 6-branes wrapped on $[c_2, d_1]$ with $2N_{f2}$ wrapped on $[d_1, c_1]$. The number of D 6-branes wrapped on $[d_1, c_1]$ remains the same (and gives the right gauge product) but we have now $2N_{f2}$ D 6-branes wrapped on $[c_2, c_1]$. The supplementary $2N_{f2}$ D 4-branes appearing in the dual picture between the NS 5-branes sitting at b and a_1 just represent $2N_{f2}$ supplementary D 6-branes wrapped on $[b, a_1]$ and they add to the $(2N_{f3} + 2N_{f2} + 2N_{f1} - N_{c2})$ D 6-branes wrapped after the transition on the same cycle. So we have $(4N_{f2} + 2N_{f3} + 2N_{f1} - N_{c2})$ D 6-branes wrapped on $[c_1, b]$ which gives the correct result for the middle gauge group. Also the final gauge group is correct when we take the dual back to the original geometrical picture. We need N_{f2} supplementary D 6-branes wrapped on $[b, a_2]$. This implies that we have N_{f2} supplementary D 6-branes wrapped on $[b, a_1]$ leading to $(3N_{f1} + 2N_{f2} + N_{f3} - N_{c1})$ D 6-branes wrapped on $[b, a_1]$ and this gives the right result for the last gauge group.

So, after transition, by wrapping N_{f2} D 6-branes on $[b, a_2]$ and by wrapping $2N_{f2}$ D 6-branes on $[c_2, c_1]$ instead of $[c_2, d_1]$ we obtain the correct magnetic theory, which has gauge group $SU(3N_{f3}+2N_{f2}+N_{f1}-N_{c3}) \times SU(4N_{f2}+2N_{f3}+2N_{f1}-N_{c2}) \times SU(3N_{f1}+2N_{f2}+N_{f3}-N_{c1})$. The additional D 6-branes came from the condition stated in the first paragraph of this section. This is the result obtained in [10] and also obtained by field theory methods.

3.2) $SU(N_{c1}) \times SU(N_{c2}) \times SU(N_{c3})$ with adjoint matter

Now we introduce adjoint matter. The electric theory has the gauge group $SU(N_{c1}) \times SU(N_{c2}) \times SU(N_{c3})$ with $N_{f1}(N_{f2})[N_{f3}]$ flavors in the fundamental representation of the first(second)[third] gauge group. Besides we have flavors in the adjoint representations of each of the components of the gauge group. Denote by $X_1(X_2)[X_3]$ the adjoint of the first(second)[third] gauge group. The superpotential which truncates the chiral ring is: $W = \mathrm{Tr}X_1^{k+1} + \mathrm{Tr}X_2^{k+1} + \mathrm{Tr}X_3^{k+1} + \mathrm{Tr}X_1 X\widetilde{X} + \mathrm{Tr}X_2 Y\widetilde{Y} + \mathrm{Tr}X_3 Y\widetilde{Y} + \mathrm{Tr}X_2 X\widetilde{X} + \rho_1 \mathrm{Tr}X_1 + \rho_2 \mathrm{Tr}X_2 + \rho_3 \mathrm{Tr}X_3$ where again ρ_i are Lagrange multipliers to enforce the tracelessness conditions for X_i. The power k can be any positive integer.

In the brane configuration picture, this corresponds to k NS 5-branes with the same orientation as A connected by N_{c1} D 4-branes with k NS 5-branes with the same orientation as B. These are connected by N_{c2} D 4-branes with k NS 5-branes with the same orientation as C and the last ones are connected by N_{c3} D 4-branes with k NS

5-branes with the same orientation as D. In the geometric picture, this means that we take k copies of all the singular points, except for a_2, c_2 and d_2. After moving c_2, d_2 to the left of all copies of a_1 and c_1, d_1 to the left of all copies of a_2, we obtain $2kN_{f2}$ D 6-branes wrapped on $[c_2, d_2]$, $(2kN_{f2}+3kN_{f3})$ D 6-branes wrapped on $[d_2, a_1]$, $(2kN_{f3}+kN_{f2}+N_{c1})$ D 6-branes wrapped on $[a_1, b]$, $(kN_{f1}+kN_{f3}+N_{c2})$ D 6-branes wrapped on $[b, c_1]$, $(2kN_{f1}+N_{c3})$ D 6-branes wrapped on $[c_1, d_1]$ and $3kN_{f1}$ D 6-branes wrapped on $[d_1, a_2]$.

We now move to the magnetic theory. The dual configuration is given by k copies of the one discussed in the previous subsection. This can be seen by explicitly exchanging the positions of b, c_1, d_1 from right to left with respect to a_1. Again we obtain $2kN_{f2}$ wrapped on $[c_2, d_1]$ and we want to extend them to D 6-branes on $[c_2, c_1]$ instead of $[c_2, d_1]$. This is required by the condition that the flavor group in the dual picture is given by D 6-branes wrapped on the pair singularity points. This is again equivalent to introducing semi-infinite D 4-branes in brane configuration picture, in order to match the field theory result. The magnetic theory has a gauge group $SU(3kN_{f3}+2kN_{f2}+kN_{f1}-N_{c3})\times SU(4kN_{f2}+2kN_{f1}+2kN_{f3}-N_{c2})\times SU(3kN_{f1}+2kN_{f2}+kN_{f3}-N_{c1})$ which coincides again with the one obtained in brane configuration picture.

3.3) Generalisation to arbitrary number of product gauge groups

Consider now the electric theory with the gauge group $SU(N_{c1})\times SU(N_{c2})\times SU(N_{c3})\times \cdots \times SU(N_{cn})$. Each gauge group has fundamental fields $N_{f1}, N_{f2}, \cdots, N_{fn}$. Besides we have the fields Y_1 in the representation $(\mathbf{N_{c1}}, \overline{\mathbf{N}}_{\mathbf{c2}}, \mathbf{1}, \cdots, \mathbf{1})$, Y_2 in the representation $(\mathbf{1}, \mathbf{N_{c2}}, \overline{\mathbf{N}}_{\mathbf{c3}}, \mathbf{1}, \cdots, \mathbf{1}), \cdots, \mathbf{Y_{n-1}}$ in the representation $(\mathbf{1}, \mathbf{1}, \cdots, \mathbf{N_{cn-1}}, \overline{\mathbf{N}}_{\mathbf{cn}})$ and their conjugate fields $\widetilde{Y}_1, \cdots, \widetilde{Y}_{n-1}$. We also need a superpotential to truncate the chiral ring.

We want to see how this looks in the geometrical picture. Take for simplicity $n=4$. Consider the singular points to be, from left to right $(a_1, c_2, b, d_2, a_2, c_1, e_2, d_1, e_1)$. We wrap N_{c1} D 6-branes on $[a_1, b]$, N_{c2} D 6-branes on $[b, c_1]$, N_{c3} D 6-branes on $[c_1, d_1]$, N_{c4} D 6-branes on $[d_1, e_1]$, N_{f1} D 6-branes on $[b, a_2]$, N_{f2} D 6-branes on $[b, c_2]$, N_{f3} D 6-branes on $[c_1, d_2]$ and N_{f4} D 6-branes on $[d_1, e_2]$ which is the electric theory. Before moving to the magnetic theory, we move c_2, d_2 and e_2 to the left of a_1 and c_1, d_1 and e_1 to the left of a_2. Some additional D 6-branes will be wrapped after each transition. We obtain, from left to right, $2N_{f2}$ D 6-branes wrapped on $[c_2, d_2]$, $(2N_{f2}+3N_{f3})$ D 6-branes wrapped on $[d_2, e_2]$, $(2N_{f2}+3N_{f3}+4N_{f4})$ D 6-branes wrapped on $[e_2, a_1]$, $(3N_{f1}+N_{c3})$ D 6-branes wrapped on $[d_1, e_1]$ and $4N_{f1}$ D 6-branes wrapped on $[e_1, a_2]$.

We now move to the magnetic theory. We push first e_1 between e_2 and a_1, so there are $\widetilde{N}_{c1}(=4N_{f4}+3N_{f3}+2N_{f2}+N_{f1}-N_{c4})$ D 6-branes wrapped on $[e_1, a_1]$. This is the right result for the first dual gauge group in the product. We subsequently move d_1, c_1 and b to the right of a_1. In the final picture, the gauge group that we obtain is not

the same as the one obtained by field theory methods. We can match it by introducing semi-infinite D 4-branes in brane configuration picture or we can match it by imposing that the D 6-branes which do not contribute to the gauge products are wrapped on 3-cycles between the pairs of points, $[a_1, a_2]$, $[c_1, c_2]$, $[d_1, d_2]$ and $[e_1, e_2]$. In this case, we have obtained the magnetic theory with the gauge group $SU(4N_{f4} + 3N_{f3} + 2N_{f2} + N_{f1} - N_{c4}) \times SU(3N_{f4} + 6N_{f3} + 4N_{f2} + 2N_{f1} - N_{c3}) \times SU(2N_{f4} + 4N_{f3} + 6N_{f2} + 3N_{f1} - N_{c2}) \times SU(N_{f4} + 2N_{f3} + 3N_{f2} + 4N_{f1} - N_{c1})$, with N_{f4} flavors in the fundamental of the first gauge group, etc. This again agrees with the results of [10] for $n = 4$.

In order to obtain a generalisation to any value of n, one has to take $(2n-1)$ singular points and to move them from right to left with respect to a reference point. We always have to be able to extend the flavor cycles to infinity. So we always push a_2 to the far right and $c_2, d_2, \cdots$ to the far left. In order to obtain the result of field theory, we have to impose that the D 6-branes which do not contribute to the gauge group are to be wrapped on 3-cycles between pairs of points, like $[a_1, a_2]$, etc. This is equivalent with introducing the semi-infinite D 4-branes in brane configuration method. Our result again agrees with the one of [10]. It is also possible to add adjoint matter fields, this imposing the multiplicity of each singular point, so the theory with adjoint is just made out of several copies of the theory without adjoint matter and the procedure is the same as before.

4 Conclusion

We have seen that a number of $N = 1$ supersymmetric field theory dualities were obtained in terms of geometric realization of wrapping D 6-branes around 3-cycles of CY threefold in type IIA string theory. The condition that the flavor gauge group in the dual theory is determined by D 6-branes wrapping around only *pairs* of singular points proved to be crucial for our construction. Our construction also gives rise to extra D 6-branes wrapping around the cycles. It would be interesting to study this transition further.

Notes added: Recent work [25] on the creation of D-branes might be related to our discussion for the supplementary D 6-branes.

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